Mathematics: The Core Course for A-level; L. Bostock and S. Chandler

Chapter 8: Differentiation II

Miscellaneous Ex 8

(1)

(a) If $y = e^{ax} . \sin bx$ Then

$$\frac{dy}{dx} = (e^{ax})' . \sin bx + e^{ax} . (\sin bx)'$$

$$= a e^{ax} . \sin bx + b e^{ax} \cos bx .$$

Hence $$\frac{d^2y}{dx^2} = (a e^{ax})' . \sin bx + a e^{ax} . (\sin bx)'$$

$$+ (b e^{ax})' . \cos bx + b e^{ax} . (\cos bx)'$$

$$= a^2 e^{ax} . \sin bx + abe^{ax} \cos bx$$

$$+ ab e^{ax} \cos bx - b^2 e^{ax} \sin bx$$

$$= e^{ax} \left((a^2 - b^2) \sin bx + 2ab \cos bx \right)$$

(b) If $y = \dfrac{1+2x}{1-2x}$ Then

$$\frac{dy}{dx} = \frac{(1-2x) . (1+2x)' - (1+2x)(1-2x)'}{(1-2x)^2}$$

So $\frac{dy}{dx} = \frac{2(1-2x) + 2(1+2x)}{(1-2x)^2}$

$= \frac{4}{(1-2x)^2}$

So $\frac{d^2y}{dx^2} = \frac{(1-2x)^2 (4)' - 4 \cdot (1-2x)^2}{(1-2x)^4}$

$= \frac{16}{(1-2x)^3}$

(c) If $x^2 - y^2 = a^2$ (assuming a is a constant)

Then $2x - 2y \cdot y' = 0$

$\therefore \frac{dy}{dx} = \frac{x}{y}$

Hence $\frac{d^2y}{dx^2} = \frac{y(x)' - x(y)'}{y^2} = \frac{y - x \cdot dy/dx}{y^2}$

$= \frac{1}{y^2}\left(y - x \cdot \frac{x}{y}\right)$

$= \frac{y^2 - x^2}{y^3}$

(2) $$\frac{d}{dx}\left(\frac{x}{1+x}\right) = \frac{(1+x)(x)' - x\cdot(1+x)'}{(1+x)^2}$$

$$= \frac{(1+x) - x}{(1+x)^2} = \frac{1}{(1+x)^2} \qquad (*)$$

Given the equation stated we have

$$\frac{d}{dx}\left(\frac{y}{1+y}\right) + \frac{d}{dx}\left(\frac{x}{1+x}\right) - \frac{d}{dx}(x^2y^3) = 0$$

$$\therefore \quad \frac{d}{dy}\left(\frac{y}{1+y}\right)\cdot\frac{dy}{dx} + \frac{d}{dx}\left(\frac{x}{1+x}\right) - \frac{d}{dx}(x^2y^3) = 0$$

By the process that gave $(*)$:

$$\frac{1}{(1+y)^2}\cdot\frac{dy}{dx} + \frac{1}{(1+x)^2} - 2xy^3 - 3x^2y^2\frac{dy}{dx} = 0$$

At $(1,1)$: $$\frac{1}{4}\frac{dy}{dx} + \frac{1}{4} - 2 - 3\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{7}{11}$$

So Equation of tangent is $y - y_0 = m(x - x_0)$

$$\therefore \quad y - 1 = -\frac{7}{11}(x-1) \Rightarrow y = -\frac{7}{11}x + 1\frac{7}{11}.$$

③ ⓐ ⓘ let $y = \frac{1}{x^2}\sqrt{1+x^3}$.

Consider $z = \sqrt{1+x^3}$ & let $u = 1+x^3$.

$\therefore \quad z = u^{1/2}$

Hence $\frac{dz}{dx} = \frac{dz}{du} \cdot \frac{du}{dx} = \frac{1}{2}u^{-1/2} \cdot (3x^2)$

$$= \frac{1}{2}(1+x^3)^{-1/2} \cdot 3x^2 \qquad \circledast$$

So $\frac{dy}{dx} = \frac{1}{x^2} \cdot \left(\sqrt{1+x^3}\right)' + \left(\frac{1}{x^2}\right)'\sqrt{1+x^3}$

$$= \frac{1}{x^2} \cdot \frac{1}{2} \cdot (1+x^3)^{-1/2} \cdot 3x^2 - \frac{2}{x^3}\sqrt{1+x^3}$$

$$= \frac{3x^2}{2x^2(1+x^3)^{1/2}} - \frac{2(1+x^3)^{1/2}}{x^3}$$

$$= \frac{3}{2\sqrt{1+x^3}} - \frac{2\sqrt{1+x^3}}{x^3}$$

$$= \frac{3x^3 - 4(1+x^3)}{2x^3(1+x^3)^{1/2}} = -\frac{x^3+4}{2x^3(1+x^3)^{1/2}}$$

ⓘⓘ let $y = \ln\left(\frac{2+\cos x}{3-\sin x}\right)$

Consider $y = \ln u$ where $u = \frac{2+\cos x}{3-\sin x}$

Then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \left(\frac{2+\cos x}{3-\sin x}\right)'$

Then $\dfrac{d}{dx}\left(\dfrac{2+\cos x}{3-\sin x}\right) = \dfrac{(3-\sin x)(2+\cos x)' - (2+\cos x)(3-\sin x)'}{(3-\sin x)^2}$

$$= \frac{-\sin x\,(3-\sin x) + \cos x\,(2+\cos x)}{(3-\sin x)^2}$$

$$= \frac{2\cos x - 3\sin x + 1}{(3-\sin x)^2}$$

$$\therefore \frac{dy}{dx} = \frac{3-\sin x}{2+\cos x} \cdot \frac{2\cos x - 3\sin x + 1}{(3-\sin x)^2}$$

$$= \frac{2\cos x - 3\sin x + 1}{(2+\cos x)(3-\sin x)}$$

(b) If $y = e^{3x}\sin 4x$

Then $\dfrac{dy}{dx} = (e^{3x})' \cdot \sin 4x + e^{3x} \cdot (\sin 4x)'$

$$= 3e^{3x} \cdot \sin 4x + 4e^{3x} \cdot \cos 4x$$

$$\therefore \frac{d^2y}{dx^2} = 9e^{3x}\sin 4x + 12e^{3x}\cos 4x$$

$$+ 12e^{3x}\cos 4x - 16e^{3x}\sin 4x$$

$$= 9e^{3x}\sin 4x + 24e^{3x}\cos 4x - 16e^{3x}\sin 4x$$

$$= -7e^{3x}\sin 4x + 24e^{3x}\cos 4x$$

So

$$-7e^{3x}\sin 4x + 24e^{3x}\cos 4x - 6(3e^{3x}\sin 4x + 4e^{3x}\cos 4x)$$

$$+ 25e^{3x}\sin 4x = 0 \quad ✓$$

(4) (a) (i) given $y = \ln(\sec 2x + \tan 2x)$

let $u = \sec 2x + \tan 2x \Rightarrow y = \ln u$.

So $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot (\sec 2x + \tan 2x)'$

Since $\sec 2x = \frac{1}{\cos 2x}$, $\frac{du}{dx} = 2 \sec 2x \tan 2x + 2 \sec^2 2x$

$$\therefore \frac{dy}{dx} = \frac{2 \sec 2x \tan 2x + 2 \sec^2 2x}{\sec 2x + \tan 2x}$$

$$= \frac{2 \sec 2x (\tan 2x + \sec 2x)}{\sec 2x + \tan 2x}$$

$$= 2 \sec 2x$$

(ii) If $y = \frac{1 + 2x^2}{1 + x^2}$ Then $\frac{dy}{dx} = \frac{(1+x^2)(1+2x^2)' - (1+2x^2)(1+x^2)'}{(1+x^2)^2}$

So $$\frac{dy}{dx} = \frac{4x(1+x^2) - 2x(1+2x^2)}{(1+x^2)^2}$$

$$= \frac{2x}{(1+x^2)^2}$$

(b) If $y = \cos\left(e^x + \frac{\pi}{4}\right)$, let $u = e^x + \frac{\pi}{4} \Rightarrow y = \cos u$.

Then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$$= -\sin u \cdot e^x = -e^x \sin\left(e^x + \frac{\pi}{4}\right) \qquad (1)$$

Repeating the same process on $\frac{dy}{dx}$ we get (using the product rule also):

$$\frac{d^2y}{dx^2} = (-e^x)' \cdot \sin\left(e^x + \frac{\pi}{4}\right) + (-e^x) \cdot \sin\left(e^x + \frac{\pi}{4}\right)'$$

$$= -e^x \cdot \sin\left(e^x + \frac{\pi}{4}\right) - e^{2x} \cos\left(e^x + \frac{\pi}{4}\right)$$

$$= -e^x \cdot \sin\left(e^x + \frac{\pi}{4}\right) - e^{2x} \cdot y \qquad (2)$$

Comparing (1) & (2) we have $\frac{d^2y}{dx^2} = \frac{dy}{dx} - e^{2x} \cdot y$

Min value is when $\frac{dy}{dx} = 0$. So by (1):

$$-e^x \cdot \sin\left(e^x + \frac{\pi}{4}\right) = 0, \text{ but } e^x > 0 \text{ always,}$$

So $\sin\left(e^x + \frac{\pi}{4}\right) = 0$. Hence $e^x + \frac{\pi}{4} = \pm n\pi$, $n = 0, 1, 2, \ldots$

$\therefore e^x = -\frac{\pi}{4} \pm n\pi$. Least positive value is $e^x = \frac{3}{4}\pi$,

hence $x = \ln\left(\frac{3\pi}{4}\right)$.

(5) (a) let $y = x^x$

$\therefore \quad \ln y = \ln x^x \quad \Rightarrow \quad \ln y = x \cdot \ln x$

Hence $\quad \frac{1}{y} \cdot \frac{dy}{dx} = \ln x + x \cdot \frac{1}{x}$

So $\quad \frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1).$

(b) let $y = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

So $\quad \tan y = \frac{1-x}{1+x}$

$$\therefore \quad \sec^2 y \cdot \frac{dy}{dx} = \frac{(1+x)(1-x)' - (1-x)(1+x)'}{(1+x)^2} = \frac{-(1+x) - (1-x)}{(1+x)^2}$$

$$= \frac{-2}{(1+x)^2}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{2}{(1+x)^2} \cdot \frac{1}{\sec^2 y} = -\frac{2}{(1+x)^2} \cdot \frac{1}{1+\tan^2 y}$$

$$= -\frac{2}{(1+x)^2\left(1+\left(\frac{1-x}{1+x}\right)^2\right)}$$

$$= -\frac{2}{(1+x)^2 + (1-x)^2} = -\frac{1}{1+x^2}$$

(6) (a) If $y = x . \ln x$ Then $\frac{dy}{dx} = (x)' . \ln x + x . (\ln x)'$

$= \ln x + 1$

For turning points, $\frac{dy}{dx} = 0$. Hence $\ln x + 1 = 0 \Rightarrow \ln x = -1$

$\therefore x = e^{-1}$ and $y = e^{-1} . \ln e^{-1} = -e^{-1}$

So coordinate is $(e^{-1}, -e^{-1})$.

To classify The turning point :

x:	$1/2$	e^{-1}	$3/4$
y':	$-$	0	$+$
	\	—	/

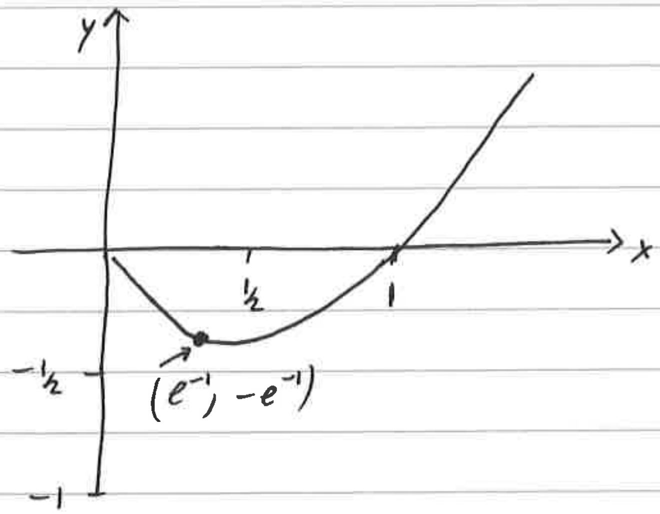
So $(e^{-1}, -e^{-1})$ is a min.

For a sketch, note that $y \to 0$ as $x \to 0$ (given)

$y = 0$ when $x = 1$

& $y \to \infty$ as $x \to \infty$

So



(b) given $x = t - \frac{1}{t}$ and $y = t + \frac{1}{t}$

we have $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

So $\frac{dy}{dt} = 1 - \frac{1}{t^2}$ and $\frac{dx}{dt} = 1 + \frac{1}{t^2}$

$\therefore \frac{dy}{dx} = \left(1 - \frac{1}{t^2}\right) \cdot \frac{1}{1 + \frac{1}{t^2}} = \frac{(t^2-1)/t^2}{(t^2+1)/t^2} = \frac{t^2-1}{t^2+1}$

For zero gradient, $\frac{dy}{dx} = 0$, hence $\frac{t^2-1}{t^2+1} = 0 \Rightarrow t^2 - 1 = 0$

$\therefore t = \pm 1$.

When $t = 1$: $x = 0$ and $y = 2$
$t = -1$: $x = 0$ and $y = -2$

So coordinates are $(0, 2)$ and $(0, -2)$

At $t = 2$: $\frac{dy}{dx} = \frac{4-1}{4+1} = \frac{3}{5}$ and $x = \frac{3}{2}$, $y = \frac{5}{2}$

Equation of tangent : $y - y_0 = m(x - x_0)$

$\Rightarrow \quad y - \frac{5}{2} = \frac{3}{5}\left(x - \frac{3}{2}\right)$

$\therefore \quad y - \frac{5}{2} = \frac{3}{5}x - \frac{9}{10} \Rightarrow y = \frac{3}{5}x - \frac{9}{10} + \frac{5}{2}$

$\therefore 5y = 3x - \frac{9}{2} + 5 = 3x + \frac{1}{2}$.

(7) given $y = \dfrac{ax+b}{cx+d}$ Then

$$\frac{dy}{dx} = \frac{(cx+d)(ax+b)' - (ax+b)(cx+d)'}{(cx+d)^2}$$

$$= \frac{a(cx+d) - c(ax+b)}{(cx+d)^2}$$

For turning points, $\dfrac{dy}{dx} = 0$, hence

$$\frac{a(cx+d) - c(ax+b)}{(cx+d)^2} = 0$$

$$\Rightarrow \quad acx + ad - acx - cb = 0$$

$$\Rightarrow \quad ad = cb \quad \Rightarrow \quad \frac{a}{b} = \frac{c}{d}$$

Substitute into y:

$$y = \frac{\frac{a}{b}x + 1}{\frac{c}{d}x + 1} = \frac{\frac{a}{b}x + 1}{\frac{a}{b}x + 1} = 1$$

and $y = 1$ has no turning point.

Now, $\dfrac{dy}{dx} = \dfrac{ad - cb}{(cx+d)^2} = \dfrac{k}{(cx+d)^2}$ where $k = ad - cb$

So $\dfrac{d^2y}{dx^2} = \dfrac{-2ck}{(cx+d)^3}$ and $\dfrac{d^3y}{dx^3} = \dfrac{6c^2k}{(cx+d)^4}$

So $$2\frac{dy}{dx}\cdot\frac{d^3y}{dx^3} = 3\left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow \quad 2\frac{k}{(cx+d)^2}\cdot\frac{6c^2k}{(cx+d)^4} = \frac{12c^2k^2}{(cx+d)^6} \quad \text{FOR LHS.}$$

and $$3\left(\frac{-2ck}{(cx+d)^3}\right)^2 = \frac{12c^2k^2}{(cx+d)^6} \quad \text{FOR RHS}$$

$\therefore$ LHS = RHS ✓

And $$2\frac{dy}{dx}\cdot\frac{d^3y}{dx^3} = 3\left(\frac{d^2y}{dx^2}\right)^2$$

(8) given $x = \sec\theta + \tan\theta$ & $y = \operatorname{cosec}\theta + \cot\theta$,

$$x + \frac{1}{x} = \sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)^2 + 1}{\sec\theta + \tan\theta} \quad (a)$$

$$= \frac{\sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta + 1}{\sec\theta + \tan\theta}$$

$$= \frac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta + \tan\theta} \quad (b)$$

because $1 + \tan^2\theta = \sec^2\theta$.

So $\quad x + \frac{1}{x} = \frac{2\sec\theta(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta} = 2\sec\theta$

as Required.

Also, $\quad y + \frac{1}{y} = \operatorname{cosec}\theta + \cot\theta + \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}{\operatorname{cosec}\theta + \cot\theta} \qquad (c)$$

$$= \frac{\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta + \cot^2\theta + 1}{\operatorname{cosec}\theta + \cot\theta}$$

$$= \frac{\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta + \operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + \cot\theta}$$

because $1 + \cot^2\theta = \operatorname{cosec}^2\theta$.

$$\therefore \quad y + \frac{1}{y} = \frac{2\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}\theta + \cot\theta}. \qquad (d)$$

$$= \frac{2\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}{\operatorname{cosec}\theta + \cot\theta}$$

$= 2\operatorname{cosec}\theta$ as Required.

Now

$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sec^2\theta$ and $\frac{dy}{d\theta} = -\cot\theta\operatorname{cosec}\theta - \operatorname{cosec}^2\theta$

So $\quad \frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = -\frac{\cot\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta}{\sec\theta\tan\theta + \sec^2\theta}$

But by the numerators of ⓐ & ⓑ & ⓒ & ⓓ

$$\frac{dy}{dx} = -\frac{\frac{1}{2}\left[(\operatorname{cosec}\theta + \cot\theta)^2 + 1\right]}{\frac{1}{2}\left[\sec\theta + \tan\theta)^2 + 1\right]} = -\frac{1+y^2}{1+x^2}$$

as required.

⑨ ⓐ given $2y e^{3x} + \frac{1}{x^2} \cdot \sin 2x = 0$

then $6y e^{3x} + 2y' e^{3x} - \frac{2}{x^3} \sin 2x + \frac{2}{x^2} \cos 2x = 0$

where $y' = \frac{dy}{dx}$.

So
$$y' = \frac{\frac{2}{x^3} \cdot \sin 2x - \frac{2}{x^2} \cdot \cos 2x - 6y e^{3x}}{2 e^{3x}}$$

$$= \frac{2 \cdot \sin 2x - 2x \cos 2x - 6x^3 y e^{3x}}{2 e^{3x} \cdot x^3}$$

$$= \frac{\sin 2x}{x^3 \cdot e^{3x}} - \frac{\cos 2x}{x^2 \cdot e^{3x}} - 3y.$$

ⓑ given $x = \frac{1+t}{1-2t}$ & $y = \frac{1+2t}{1-t}$ Then

$$\frac{dx}{dt} = \frac{(1-2t)(1+t)' - (1+t)(1-2t)'}{(1-2t)^2}$$

$$= \frac{(1-2t) + 2(1+t)}{(1-2t)^2} = \frac{3}{(1-2t)^2}$$

and $\frac{dy}{dt} = \frac{(1-t)(1+2t)' - (1+2t)(1-t)'}{(1-t)^2}$

$$= \frac{2(1-t) + (1+2t)}{(1-t)^2} = \frac{3}{(1-t)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{3}{(1-2t)^2} \cdot \frac{3}{(1-t)^2}$$

when $t=0$, $\frac{dy}{dx} = 9$.

(c) given $f(x) \equiv 3x + \sin x - 8 \sin \frac{1}{2} x$

Then $\frac{df}{dx} = 3 + \cos x - 4 \cos \frac{1}{2} x$

If $f(x) > 0$ for $x > 0$ Then $f'(x) \geqslant 0$

So $3 + \cos x - 4 \cos \frac{1}{2} x \geqslant 0$

$\Longrightarrow 3 + \cos x \geqslant 4 \cos \frac{1}{2} x$

Max positive value of cos is $+1$, So $3+1 \geqslant 4$ ✓
Min positive value of cos is 0, So $3+0 \geqslant 0$ ✓

So $f(x)$ is Positive for $x > 0$

(10) Let $y = \sin^{-1}x$. Then $\sin y = x$

$\therefore \quad \cos y \cdot \dfrac{dy}{dx} = 1$ by implicit differentiation

So $\dfrac{dy}{dx} = \dfrac{1}{\cos y}$

But $\sin y = x \Rightarrow 1 - \sin^2 y = 1 - x^2 = \cos^2 y$

So $\cos y = \pm\sqrt{1-\sin^2 y} = \sqrt{1-x^2}$

$\therefore \quad \dfrac{dy}{dx} = \pm\dfrac{1}{\sqrt{1-x^2}}$ (*)

However $y = \sin^{-1}x$ has positive gradient, $\therefore$ choose the positive square root in (*).

So $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

By the same process we can show that $\dfrac{d}{dx}(\sin^{-1}2x) = \dfrac{2}{\sqrt{1-4x^2}}$

By implicit differentiation we can show that $\dfrac{d}{dx}(\sin^{-1}y) = \dfrac{1}{\sqrt{1-y^2}} \cdot \dfrac{dy}{dx}$

For $\arcsin(xy)$, let $z = \sin^{-1}x.y$. Then $\sin z = xy$

$\therefore \dfrac{dz}{dx} \cdot \cos z = y + x \cdot \dfrac{dy}{dx}$

So $\dfrac{dz}{dx} = \dfrac{y + x \cdot dy/dx}{\cos z} = \pm \dfrac{y + x \cdot dy/dx}{\sqrt{1 - \sin^2 z}}$

$= + \dfrac{y + x \cdot dy/dx}{\sqrt{1 - (xy)^2}}$

So $\sin^{-1}(2x) + \sin^{-1} y + \sin^{-1}(xy) = 0$ we have

$$\frac{2}{\sqrt{1-4x^2}} + \frac{1}{\sqrt{1-y^2}} \cdot \frac{dy}{dx} + \frac{y + x \cdot dy/dx}{\sqrt{1-(xy)^2}} = 0$$

when $x = y = 0$ we have

$$2 + \frac{dy}{dx} + 0 = 0 \implies \frac{dy}{dx} = -2.$$

(11) Not a differentiation question. would be more suitable as a numerical methods question, using Bisection & Newton-Raphson. Newton-Raphson involves differentiation, but This chapter does not go Through This method.

(12) (a) given $y = \sqrt{5x^2+3}$, let $u = 5x^2+3$, & $y = \sqrt{u}$.

Then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{2} \dfrac{1}{\sqrt{u}} \cdot (10x)$

$= \dfrac{1}{2\sqrt{5x^2+3}} \cdot (10x)$

$= \dfrac{5x}{(5x^2+3)^{1/2}}$

So $\dfrac{d^2y}{dx^2} = \dfrac{(5x^2+3)^{1/2}(5x)' - (5x).[(5x^2+3)^{1/2}]'}{5x^2+3}$

$= \dfrac{5(5x^2+3)^{1/2} - (5x)(1/2)(10x)(5x^2+3)^{-1/2}}{5x^2+3}$

$= \dfrac{5(5x^2+3) - 25x^2}{(5x^2+3)^{3/2}} = \dfrac{15}{(5x^2+3)^{3/2}}$

So $y.\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = (5x^2+3)^{1/2}.\dfrac{15}{(5x^2+3)^{3/2}} + \dfrac{25x^2}{5x^2+3}$

$= \dfrac{15}{5x^2+3} + \dfrac{25x^2}{5x^2+3}$

$= \dfrac{5(5x^2+3)}{5x^2+3} = 5$ ✓

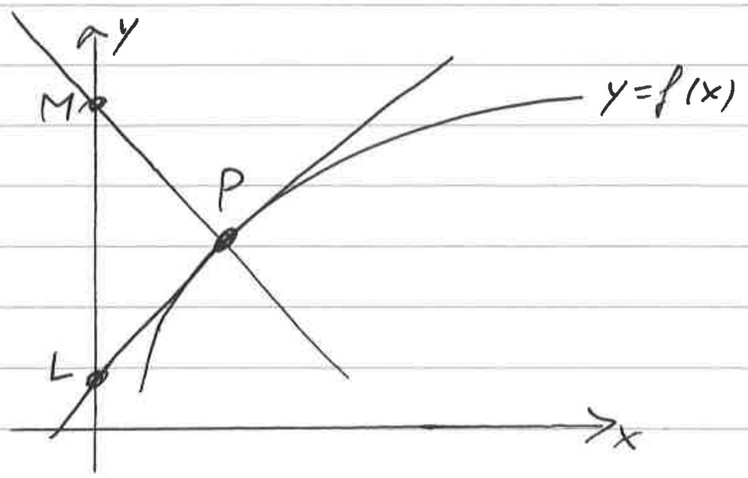
(b) given $x = 3(2\theta - \sin 2\theta)$ and $y = 3(1 - \cos 2\theta)$

we have $\dfrac{dx}{d\theta} = 6 - 6\cos 2\theta$, $\dfrac{dy}{d\theta} = 6\sin 2\theta$

So $\dfrac{dy}{dx} = \dfrac{dy}{d\theta}.\dfrac{d\theta}{dx} = \dfrac{6\sin 2\theta}{6 - 6\cos 2\theta} = \dfrac{\sin 2\theta}{1-\cos 2\theta}$.

NORmal : $m_1 m_2 = -1 \implies \dfrac{dx}{dy} = -\dfrac{1-\cos 2\theta}{\sin 2\theta}$.

At $\theta = \frac{\pi}{4}$: $\dfrac{dy}{dx} = 1 \implies \dfrac{dx}{dy} = -1$



Now, at $\theta = \frac{\pi}{4}$ we have $x = \frac{3\pi}{2} - 3$ and $y = 3$

So Point $P(x, y) = P\left(\frac{3\pi}{2} - 3,\ 3\right)$

$\therefore$ Equation of tangent y_t is $y_t - 3 = 1.\left(x - \frac{3\pi}{2} + 3\right)$

$$\Rightarrow \quad y_t = x - \frac{3\pi}{2} + 6$$

& Equation of Normal y_n is $y_n - 3 = -1\left(x - \frac{3\pi}{2} + 3\right)$

$$\Rightarrow \quad y_n = -x + \frac{3\pi}{2}$$

At $x = 0$, $y_t = -\frac{3\pi}{2} + 6$ and $y_n = \frac{3\pi}{2}$

So $|LM| = \frac{3\pi}{2} - \left(-\frac{3\pi}{2} + 6\right) = 3\pi - 6$

The height to P is x value when $\theta = \frac{\pi}{4}$, so Δ height $= \frac{3\pi}{2} - 3$

Area A of Δ is $\therefore$ $A = \frac{1}{2}(3\pi - 6)\left(\frac{3\pi}{2} - 3\right) = \left(\frac{3\pi}{2} - 3\right)^2$

$$= \frac{9\pi^2}{4} - 2 \cdot \frac{9\pi}{2} + 9 = \frac{9}{4}(\pi - 2)^2 \checkmark$$

(13) If $y = \dfrac{2x^2}{(2x-1)(x+1)} = \dfrac{2x^2}{2x^2+x-1}$

By long division:

$$
\begin{array}{r|l}
 & 1 \\
2x^2+x-1 & 2x^2 \\
 & 2x^2+x-1 \\
\hline
 & -x+1
\end{array}
$$

So $\quad y = 1 + \dfrac{1-x}{(2x-1)(x+1)} \quad (*)$

Convert The Rational f of $(*)$ into Partial fractions:

$$\frac{1-x}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$$

So $\quad 1-x = A(x+1) + B(2x-1)$

if $x=-1$: $\quad 2 = -3B \implies B = -\frac{2}{3}$

if $x=\frac{1}{2}$: $\quad \frac{1}{2} = \frac{3}{2}A \implies A = \frac{1}{3}$

So $\quad y = 1 + \frac{1}{3}\cdot\dfrac{1}{2x-1} - \frac{2}{3}\cdot\dfrac{1}{x+1}$

$$\therefore \frac{dy}{dx} = \frac{1}{3}(-1)(2)\frac{1}{(2x-1)^2} + \frac{2}{3}\frac{1}{(x+1)^2}$$

$$= -\frac{2}{3}\frac{1}{(2x-1)^2} + \frac{2}{3}\frac{1}{(x+1)^2}$$

$$\therefore \quad \frac{d^2y}{dx^2} = -\frac{2}{3}(-2)(2)\frac{1}{(2x-1)^3} + \frac{2}{3}(-2)\frac{1}{(x+1)^3}$$

$$= \frac{8}{3} \cdot \frac{1}{(2x-1)^3} - \frac{4}{3} \cdot \frac{1}{(x+1)^3}$$

At $x=1$, $\frac{d^2y}{dx^2} = \frac{15}{6} = \frac{5}{2}$

(14) (a) let $y_1 = \ln(k\sec x)$ & $y_2 = a^x$

For y_1: let $u = k\sec x$, $\therefore y_1 = \ln u$

hence $\frac{dy_1}{dx} = \frac{dy_1}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot k\sec x \cdot \tan x$

$= \tan x$

For y_2: $\ln y_2 = x \ln a$

So $\frac{1}{y_2} \cdot \frac{dy_2}{dx} = \ln a \Rightarrow \frac{dy_2}{dx} = y_2 \ln a$

$= a^x \ln a$

So $\frac{dy}{dx} = \tan x + a^x . \ln a$

(b) given $x = \sin t$ and $y = \cos 2t$ Then

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = -2\sin 2t \cdot \frac{1}{\cos t} = -2 \cdot \frac{2\sin t \cdot \cos t}{\cos t}$

$= -4\sin t.$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(-4 \cdot \sin t\right) \cdot \frac{1}{\cos t}$$

$$= -4\cos t \cdot \frac{1}{\cos t} = -4$$

So $\frac{d^2y}{dx^2} + 4 = -4 + 4 = 0$ ✓

(c) Let $y = \frac{x-3}{x^2-x-2}$

Then $\frac{dy}{dx} = \frac{(x^2-x-2)\cdot 1 - (x-3)(2x-1)}{(x^2-x-2)^2}$

For stationary points $\frac{dy}{dx} = 0$, hence

$$\frac{x^2-x-2-(x-3)(2x-1)}{(x^2-x-2)^2} = 0$$

$\therefore \quad x^2 - x - 2 - (2x^2 - 7x + 3) = 0$

$\Rightarrow \quad -x^2 + 6x - 5 = 0$

i.e $\quad x^2 - 6x + 5 = 0$

$\therefore \quad (x-5)(x-1) = 0$

$\therefore \quad x = 1, 5$

x : 0 1 2 5 6

y' : \ — / — \

Also at $x=1$, $y=1$ and at $x=5$, $y=\frac{1}{9}$

So min at $y=1$ & max at $y=\frac{1}{9}$

(15) given $x^2y = a\cos nx$ (n = constant)

Then $2xy + x^2.\frac{dy}{dx} = -na.\sin nx$ ⊛

and $2y + 2x.\frac{dy}{dx} + 2x.\frac{dy}{dx} + x^2.\frac{d^2y}{dx^2} = -n^2a\cos nx$ ⊛⊛

By ⊛ $\frac{dy}{dx} = \frac{-2xy - na.\sin nx}{x^2}$

So substituting into ⊛⊛ :

$$2y + 4x\left(\frac{-2xy - na\sin nx}{x^2}\right) + x^2.\frac{d^2y}{dx^2} = -n^2a\cos nx$$

$$= -n^2ay$$

$$\therefore \quad x^2.\frac{d^2y}{dx^2} + 4x.\frac{dy}{dx} + (n^2a+2)y = 0$$

[The "x^2" in the Brackets is NOT correct]

(b) given $x = t^2 + 3$, $y = t(t^2+3)$

i) To show symmetry about x-axis replace y by $-y$ in the "x" equation:

Firstly: $y = xt \Rightarrow \frac{y}{x} = t$

$\therefore \quad x = \frac{y^2}{x^2} + 3 \qquad (1)$

For symmetry: $x = \frac{(-y)^2}{x^2} + 3 = \frac{y^2}{x^2} + 3 \qquad (2)$

Since (1) & (2) are the same the curve is symmetric about the x-axis.

ii) By (2) $\frac{y^2}{x^2}$ is always greater than or equal to zero (i.e. always positive), so $\frac{y^2}{x^2} + 3$ is always greater than or equal to 3.

Another way to see this is to solve $t^2 + 3 < 3$.

$\therefore \quad t^2 < 0$, i.e t^2 is negative, but this can't be so, (we can't take roots of negative N°s)

$\therefore \quad x \geq 3$.

iii) $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = (3t^2+3) \cdot \frac{1}{2t} = \frac{3}{2}t + \frac{3}{2t}$.

So $\dfrac{dy}{dx} = \left(\frac{1}{2}t + \frac{1}{2t}\right)$

So $\left(\dfrac{dy}{dx}\right)^2 = 9\left(\frac{1}{2}t + \frac{1}{2t}\right)^2$

Test $\left(\dfrac{dy}{dx}\right)^2 \geqslant 9$: $\quad 9\left(\frac{1}{2}t + \frac{1}{2t}\right)^2 \geqslant 9$

$$\Rightarrow \quad \frac{1}{2}t + \frac{1}{2t} \geqslant \pm 1$$

For "+1" : $\frac{1}{2}t + \frac{1}{2t} \geqslant 1 \Rightarrow t^2 + 1 \geqslant 2t$

$$\Rightarrow t^2 - 2t + 1 \geqslant 0$$

$$\Rightarrow (t-1)^2 \geqslant 0 \text{ is true}$$

For "−1" : $\frac{1}{2}t + \frac{1}{2t} \geqslant -1 \Rightarrow t^2 + 1 \geqslant -2t$

$$\Rightarrow t^2 + 2t + 1 \geqslant 0$$

$$\Rightarrow (t+1)^2 \geqslant 0 \text{ is true.}$$

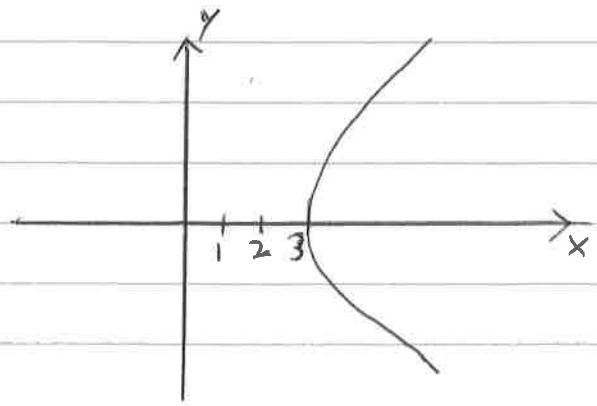
So $\left(\dfrac{dy}{dx}\right)^2 \geqslant 9$ is true.

Sketch

At $x = 3$, $3 = t^2 + 3 \Rightarrow t = 0$

$\Rightarrow \dfrac{dy}{dx} \to \infty$, i.e vertical tangent.

As $t \to +\infty$, $x \to +\infty$ and $y \to +\infty$



(16) (a) i) Let $y = \ln\left(x + \sqrt{x^2+1}\right)$.

Now let $u = x + \sqrt{x^2+1}$, $\therefore y = \ln u$.

Hence $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \left(1 + \frac{1}{2}(2x)(x^2+1)^{-1/2}\right)$

$$= \frac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}}$$

$$= \frac{\left(\sqrt{x^2+1} + x\right) / \sqrt{x^2+1}}{x + \sqrt{x^2+1}}$$

$$= \frac{1}{\sqrt{x^2+1}}$$

ii) Let $y = \sec^2 2x$.

Now let $u = \sec 2x$, $\therefore y = u^2$.

So $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot \left(2\sec 2x \cdot \tan 2x\right)$

$= 2\sec 2x\left(2\sec 2x \cdot \tan 2x\right)$

$= 4\sec^2 2x \cdot \tan 2x$

iii) Let $y = 10^{3x}$, so $\ln y = 3x \ln 10$

$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = 3\ln 10$; hence $\frac{dy}{dx} = 3y \cdot \ln 10 = 3 \cdot 10^{3x} \cdot \ln 10$

(b) given $x^2 + y^2 = 2y$, $\circledast$

Then $2x + 2y \cdot \frac{dy}{dx} = 2\frac{dy}{dx}$

$\therefore \quad (2y - 2)\frac{dy}{dx} = -2x$

Hence $\frac{dy}{dx} = \frac{2x}{2-2y} = \frac{x}{1-y}$

$$\therefore \quad \frac{d^2y}{dx^2} = \frac{(1-y)\cdot(x)' - x\cdot(1-y)'}{(1-y)^2}$$

$$= \frac{1-y - x(-dy/dx)}{(1-y)^2}$$

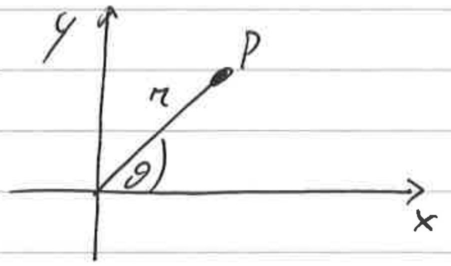
$$= \frac{1-y + x(x/(1-y))}{(1-y)^2}$$

$$= \frac{(1-y)^2 + x^2}{(1-y)^3} = \frac{(1-y)^2 + 2y - y^2}{(1-y)^3} \quad \text{by } \circledast$$

$$= \frac{1}{(1-y)^3}$$

(17) A Sketch of The Relevant issue

(a)



$$r^2 = x^2 + y^2$$

$$= e^{-2t} + 4t^2 e^{-2t}$$

$$\therefore \frac{d}{dt}(r^2) = -2e^{-2t} + 8t\,e^{-2t} - 8t^2 e^{-2t}$$

$$= -2e^{-2t}(1 - 4t + 4t^2)$$

$$= -2e^{-2t}(1-2t)^2$$

(b) $\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2t.e^{-t}}{e^{-t}} = \tan^{-1} 2t$

So $\tan\theta = 2t \Rightarrow \sec^2\theta . \frac{d\theta}{dt} = 2$

$$\therefore \frac{d\theta}{dt} = \frac{2}{\sec^2\theta} = \frac{2}{1+\tan^2\theta} = \frac{2}{1+4t^2}$$

Note That in (a) we treat r^2 as a complete / Stand alone Variable, Not as The square of r. I.e. we do not use implicit differentiation on r^2. Another way of saying This is That we could call $r^2 = Z$, & find $\frac{dZ}{dt}$.

(18) given $y = x.\tan^{-1}x$, Then $\tan y/x = x$

So (a) $\frac{1}{x^2}\left(x\cdot\frac{dy}{dx} - y\right)\cdot\sec^2\frac{y}{x} = 1$ by implicit differentiation & The quotient Rule

$$\therefore \quad x\cdot\frac{dy}{dx} - y = \frac{x^2}{\sec^2 y/x}$$

$$= \frac{x^2}{1+\tan^2\frac{y}{x}} = \frac{x^2}{1+x^2}$$

$$\therefore \quad x(1+x^2)\frac{dy}{dx} - y(1+x^2) = x^2$$

$$\Rightarrow \quad x(1+x^2)\frac{dy}{dx} = x^2 + y(1+x^2) \qquad (*)$$

(b) Differentiate (*) to get

$$(1+3x^2)\frac{dy}{dx} + x(1+x^2)\frac{d^2y}{dx^2} = 2x + \frac{dy}{dx}(1+x^2) + y.2x$$

$$\text{So} \quad x(1+x^2)\frac{d^2y}{dx^2} + 2x^2\frac{dy}{dx} = 2x(1+y)$$

$$\text{So} \quad (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 2$$

Sketch: There is No way of knowing The Shape of The curve $y = x.\tan^{-1}x$ FROM (a) & (b) UNless you do a proper curve sketching analysis, which is only taught in chap 11, not This chapter. So I don't see how a sketch can be given.

(19) given $\tan y = x$ Then

$$\frac{dy}{dx} \cdot \sec^2 y = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y}$$

$$= \frac{1}{1+x^2}$$

$$\therefore \frac{d^2y}{dx^2} = (2x)(-1)\,(1+x^2)^{-2}$$

$$= -\frac{2x}{(1+x^2)^2}$$

when $y = \frac{\pi}{4}$, $\tan\frac{\pi}{4} = x = 1$. Hence

$$\frac{d^2y}{dx^2} = -\frac{2}{2^2} = -\frac{1}{2}.$$

(20) (a) i) let $y = x^2 \cdot \sin 3x$

$$\therefore \frac{dy}{dx} = 2x \cdot \sin 3x + x^2(3\cos 3x)$$

$$= x(2\sin 3x + 3x\cos 3x)$$

ii) let $y = e^{-2/x}$ & let $u = -\frac{2}{x} \Rightarrow y = e^u$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot \left(+\frac{2}{x^2}\right) = \frac{2}{x^2} e^{-2/x}$$

iii) Let $y = \left(\frac{x-1}{2-x}\right)^2$

$$\therefore \ln y = 2\ln(x-1) - 2\ln(2-x)$$

$$\Rightarrow \quad \frac{1}{y} \cdot \frac{dy}{dx} = \frac{2}{x-1} + \frac{2}{2-x}$$

$$\therefore \quad \frac{dy}{dx} = \left(\frac{x-1}{2-x}\right)^2 \left(\frac{2(2-x) + 2(x-1)}{(x-1)(2-x)} \right)$$

$$= \left(\frac{x-1}{2-x}\right)^2 \left(\frac{2}{(x-1)(2-x)} \right)$$

$$= \frac{2(x-1)}{(2-x)^3}$$

(we could have used chain rule & quotient rule but logs were easier here)

(b) given $y = \ln(1 + \sin x)$, Let $u = 1 + \sin x$, $\Rightarrow y = \ln u$.

$$\therefore \quad \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot (\cos x)$$

$$= \frac{\cos x}{1 + \sin x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{(1 + \sin x) \cdot (\cos x)' - \cos x \cdot (1 + \sin x)'}{(1 + \sin x)^2}$$

$$= \frac{-(1 + \sin x) \cdot \sin x - \cos x (\cos x)}{(1 + \sin x)^2}$$

$$= - \frac{\sin x + \sin^2 x + \cos^2 x}{(1 + \sin x)^2} = - \frac{1}{(1 + \sin x)}$$

So $\frac{d^2y}{dx^2} = -e^{-y} \Rightarrow \frac{d^2y}{dx^2} + e^{-y} = 0$

Stationary Values occur at $\frac{dy}{dx} = 0$. (Note That $\frac{d^2y}{dx^2} = 0$ Does not imply Stationary Values).

So $\frac{\cos x}{1+\sin x} = 0 \Rightarrow \cos x = 0$

$\Rightarrow x = \frac{\pi}{2} \pm n\pi$

<u>Test a few Values</u>

For $x = \frac{\pi}{2}$, $\frac{\cos \pi/2}{1+\sin \pi/2} = \frac{0}{2} = 0$

For $x = \frac{3\pi}{2}$, $\frac{\cos 3\pi/2}{1+\sin \frac{3\pi}{2}} = \frac{0}{1-1} = \frac{0}{0}$ Not Possible

The above Repeats for $x = \frac{5\pi}{2}$ and $x = \frac{7\pi}{2}$

$x = \frac{9\pi}{2}$ and $x = \frac{11\pi}{2}$ etc...

<u>Classify</u> : x : $\frac{\pi}{4}$ $\frac{\pi}{2}$ $\frac{3\pi}{4}$

y' : +ve 0 −ve

So y is max at $x = \frac{\pi}{2}$ and $\therefore$ at $x = \frac{5\pi}{2}$ etc...

$\Rightarrow$ All Stationary points are max.

(21) See your own textbook for a proof of the quotient Rule.

Let $y = \dfrac{2+\ln(1+x)^2}{2-\ln(1-x)^2}$ Then

Now let $y_1 = 2+\ln(1+x)^2 = 2-2\ln(1+x)$

$\therefore \dfrac{dy_1}{dx} = \dfrac{2}{1+x}$

Let $y_2 = 2-\ln(1-x)^2 = 2-2\ln(1-x)$

$\therefore \dfrac{dy_2}{dx} = +\dfrac{2}{1-x}$

So $$\frac{dy}{dx} = \frac{[2-\ln(1-x)^2]\cdot\frac{2}{1+x} - [2+\ln(1+x)^2]\cdot\frac{+2}{1-x}}{[2-\ln(1-x)^2]^2}$$

$$= \frac{2[2-\ln(1-x)^2](1-x) - 2[2+\ln(1+x)^2](1+x)}{(1+x)(1-x)[2-\ln(1-x)^2]^2}$$

$$= \frac{2[2-2\ln(1-x)](1-x) - 2[2+2\ln(1+x)](1+x)}{(1+x)(1-x)[2-2\ln(1-x)]^2}$$

$$= \frac{4(1-x) - 4(1-x)\ln(1-x) - 4(1+x) - 4(1+x)\ln(1+x)}{(1+x)(1-x)[2-2\ln(1-x)]^2}$$

$$= \frac{-2x - (1-x)\ln(1-x) - (1+x)\ln(1+x)}{(1+x)(1-x)[1-\ln(1-x)]^2}.$$

(22) given $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$

Then $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} = a\sin\theta \cdot \frac{1}{a - a\cos\theta}$

$$= \frac{\sin\theta}{1-\cos\theta}$$

Now, $\sin\theta = \sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right) = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$

& $\cos\theta = \cos 2\cdot\left(\frac{\theta}{2}\right) = 1 - 2\sin^2\frac{\theta}{2}$ (could have used $2\cos^2\frac{\theta}{2} - 1$ or $\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$)

So $\frac{dy}{dx} = \frac{2\sin\theta/2 \cdot \cos\theta/2}{1-\left(1-2\sin^2\frac{\theta}{2}\right)}$

$$= \frac{\cos\theta/2}{\sin\theta/2} = \cot\tfrac{1}{2}\theta$$

At $\theta = \frac{1}{2}\pi$: $x = a\left(\frac{\pi}{2}-1\right)$ & $y = a$

$\therefore$ Point A has coordinate $\left(a\left(\frac{\pi}{2}-1\right), a\right)$

At $\theta = \frac{3\pi}{2}$: $x = a\left(\frac{3\pi}{2}+1\right)$ & $y = a$

$\therefore$ Point B has coordinate $\left(a\left(\frac{3\pi}{2}+1\right), a\right)$

At $\theta = \frac{\pi}{2}$, $\frac{dy}{dx} = \cot\frac{\pi}{4} = 1$

$\therefore$ Equation of tangent at A is

$$y - a = 1.\left[x - a\left(\frac{\pi}{2} - 1\right)\right]$$

So $y = x - a\frac{\pi}{2} + 2a$

At $\theta = \frac{3\pi}{2}$, $\frac{dy}{dx} = \cot\frac{3\pi}{4} = -1$

$\therefore$ Equation of tangent at B is

$$y - a = -1.\left[x - a\left(\frac{3\pi}{2} + 1\right)\right]$$

$$\therefore \quad y = -x + a.\frac{3\pi}{2} + 2a$$

(23) Let $y = \arccos x$. Hence $\cos y = x$

$$\therefore \quad \frac{dy}{dx}.(-\sin y) = 1$$

So $\frac{dy}{dx} = -\frac{1}{\sin y} = \frac{-1}{\pm\sqrt{1-\cos^2 y}} = \frac{-1}{\pm\sqrt{1-x^2}}$

Question says we look only at Positive SquareRoot, $\therefore$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

So $$\frac{df}{dx} = \frac{-1}{\sqrt{1-x^2}} - (1-x^2)^{\frac{1}{2}} - x\left(\tfrac{1}{2}\right)(-2x)(1-x^2)^{-\frac{1}{2}}$$

where the last term was found by the chain rule.

So $$\frac{df}{dx} = -\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}$$

$$= \frac{x^2-1}{\sqrt{1-x^2}} - \sqrt{1-x^2}$$

$$= \frac{x^2-1-(1-x^2)}{\sqrt{1-x^2}} = \frac{-2-2x^2}{\sqrt{1-x^2}}$$

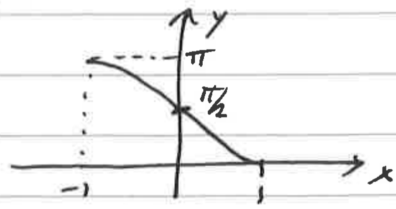
$$= -2\sqrt{1-x^2}\ . \quad \circledast$$

If $f(x)$ is decreasing then $\frac{df}{dx}$ has to be negative, which it is by $\circledast$

Since $-1 < x < 1$, the square root can always be evaluated & will always be positive.

Sketch: At $x=-1$, $f(-1)=\pi$; at $x=1$, $f(1)=0$;

At $x=0$, $f(0)=\pi/2$. Also since $f(x)$ decreases from $-1<x<1$ $f(x)$ is max at $x=-1$ & $f(x)$ is min at $x=1$. Hence



24 This is a trig question not a differentiation question.